

JEE QUESTION: MOTION IN A LINE

- 1) Two bodies begin to fall freely from the same height. The second body begins to fall ' τ ' sec. After the first. After what time from the beginning of first body does the distance between the bodies become equal to l ?

ANSWER

Let the time of fall of the 1st body be t seconds

Time of fall of 2nd body = $t - \tau$

Distance of free fall of the bodies are

$$H_1 = \frac{gt^2}{2} ; H_2 = \frac{g(t - \tau)^2}{2}$$

$$\text{Therefore, } l = H_1 - H_2 = \frac{gt^2}{2} - \frac{g(t - \tau)^2}{2}$$

$$l = gt\left(\tau - \frac{t}{2}\right)$$

$$\Rightarrow t = \frac{l}{g\tau} + \frac{\tau}{2}$$

- 2) Two PARALLEL RAIL TRACKS RUN NORTH SOUTH. TRAIN A MOVES NORTH WITH A SPEED OF 54 km/h AND TRAIN B MOVES SOUTH WITH A SPEED OF 90 km/h. What is

a) velocity of B w.r.t A

b) velocity of ground w.r.t B

c) velocity of monkey running on the roof of train A against its motion (with a velocity of 18 km/h w.r.t A) as observed by a man standing on the ground

ANSWER

$$v_A = +54 \text{ km/h} = 15 \text{ m/s} , v_B = -90 \text{ km/h} = -25 \text{ m/s}$$

$$\begin{aligned} \text{i) } v_{BA} &= v_B - v_A = -25 - 15 \\ &= -40 \text{ m/s} \end{aligned}$$

i.e, the train B appears to A to move with a speed of 40 m/s from north to south

$$\begin{aligned}
 \text{ii) } v_{GB} &= v_G - v_B \\
 &= 0 - (-25) \\
 &= 25 \text{ m/s}
 \end{aligned}$$

iii) Let the velocity of the monkey w.r.t ground be v_M . Relative velocity of the monkey w.r.t to A,

$$\begin{aligned}
 v_{MA} &= v_M - v_A \\
 &= -18 \text{ km/h} \\
 &= -5 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 v_M &= v_{MA} + v_A = (-5 + 15) \\
 &= 10 \text{ m/s}
 \end{aligned}$$

3. If a freely falling body covers half of its total distance in the last second. Find its time of fall.

ANSWER

Suppose, t is the time

$$h = \frac{1}{2}gt^2 \quad (1)$$

$$\frac{h}{2} = \frac{1}{2}g(t-1)^2 \quad (2)$$

Solving ① & ②

$$\frac{1}{4}gt^2 = \frac{1}{2}g(t^2 + 1 - 2gt)$$

$$\frac{1}{2}t^2 = gt^2 - 2gt + g$$

$$t = \underline{\underline{(2+\sqrt{2})\text{sec}}}$$

Q1. A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1 . If it is projected vertically downwards from the same point with the speed being same, it reaches the ground in t_2 . Time required to reach the ground, if it is dropped from the top of the tower is:

A: h = height of the tower

u = initial speed of projection (same magnitude in both cases)

g = acceleration due to gravity (positive downwards)

t_1 = Time taken to reach the ground when projected upwards

t_2 = Time taken to reach the ground when projected downwards

t = time taken to reach ground when simply dropped

Case: 1 - Projectile thrown upwards

initial velocity = $-u$ [thrown upwards]

time taken = t_1

$$h = -ut_1 + \frac{1}{2}gt_1^2$$

Case: 2 - Projectile thrown downwards

initial velocity = u

time taken = t_2

$$h = ut_2 + \frac{1}{2}gt_2^2$$

Case: 3 - Body simply dropped

$u = 0$

t = time taken

$$h = \frac{1}{2}gt^2$$

On equating the heights obtained from case 1 and 2

$$-ut_1 + \frac{1}{2}gt_1^2 = ut_2 + \frac{1}{2}gt_2^2$$

$$-ut_1 - ut_2 = \frac{1}{2}gt_2^2 - \frac{1}{2}gt_1^2$$

$$-u(t_1 + t_2) = \frac{1}{2}g(t_2^2 - t_1^2)$$

$$-u(t_1 + t_2) = \frac{1}{2}g(t_2 - t_1)(t_2 + t_1)$$

$$-u = \frac{1}{2}g(t_2 - t_1)$$

$$u = \frac{1}{2}g(t_1 - t_2)$$

On expressing h in terms of t_1 and t_2

$$h = -\left(\frac{1}{2}g[t_1 - t_2]\right)t_1 + \frac{1}{2}gt_1^2$$

$$h = \frac{1}{2}gt_1t_2$$

equating the height for dropped case:

$$h = \frac{1}{2}gt^2 \text{ [dropped case]}$$

$$\frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2$$

$$t^2 = t_1t_2$$

$$t = \sqrt{t_1t_2}$$

$\therefore$ The time required for the body to reach the ground when dropped is the geometric mean of t_1 and t_2

Q2. The position of a particle related to time is given by $x = (5t^2 - 4t + 5)\text{m}$. The magnitude of velocity of the particle at $t = 2\text{s}$ will be:

A: The position of a particle as a function of time is given by

$$x = (5t^2 - 4t + 5)\text{m}$$

to find magnitude of the velocity of the particle at $t = 2\text{s}$, we first need to find the velocity of the particle as a function of time.

The velocity v is the time derivative of the position ' x '

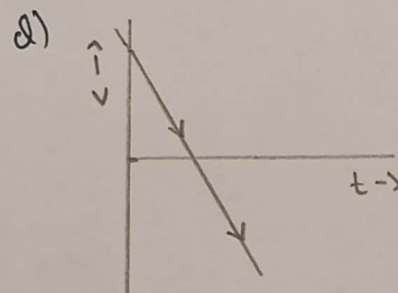
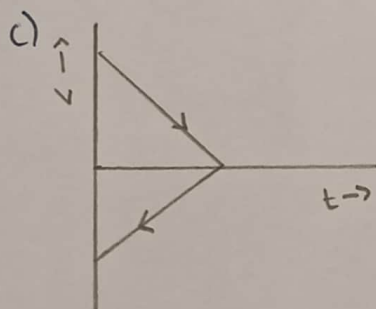
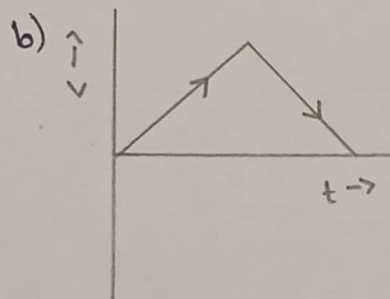
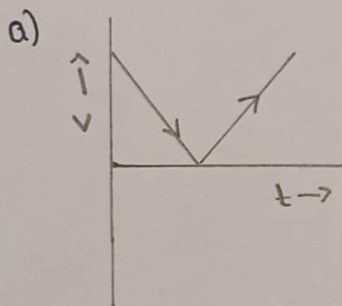
$$v = \frac{dx}{dt} \Rightarrow v = \frac{dx}{dt} = 10t - 4 \text{ m/s}$$

Now we can find the velocity of the particle at $t=2s$ by plugging $t=2$

$$v(2s) = 10(2) - 4m/s = 16m/s$$

$\therefore$ The magnitude of the velocity of the particle at $t=2s$ is: $|v(2s)| = 16m/s$

Q3. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time



A: Initially the speed of particle is maximum (V_0) and at height h velocity will become zero, then particle will move downward and velocity will increase gradually and become maximum when it reaches the ground and acceleration in this entire motion will be $-g$, so slope of $v-t$ will be same and negative

$\therefore$ Option d) is correct

JEE Questions

Motion In A Straight Line

①

Q.1) A particle is moving in a straight line. The variation of position 'x' as a function of time 't' is given as

$x = (t^3 - 6t^2 + 20t + 15) \text{ m}$. The velocity of the body when its acceleration becomes zero is. [JEE Mains 2025]

Ans:- $x = (t^3 - 6t^2 + 20t + 15) \text{ m}$

$$\frac{d(x)}{dt} = \frac{d}{dt}(3t^2 - 12t + 20 + 0) \text{ m} \quad \text{--- (1)}$$

$$\frac{dv}{dt} = \frac{d}{dt}(6t - 12) \text{ m}$$

$$a = 6t - 12 \quad [\because a = 0]$$

$$\therefore t = \frac{12}{6} \Rightarrow \underline{\underline{2 \text{ sec}}} \quad \text{--- (2)}$$

$$\text{Sub } t = 2 \text{ sec in (1)}$$

$$\Rightarrow 3t^2 - 12t + 20$$

$$\Rightarrow 3(2)^2 - 12(2) + 20$$

$$\Rightarrow 12 - 24 + 20$$

$$\Rightarrow -12 + 20$$

$$\Rightarrow \underline{\underline{8 \text{ m/s}}}$$

Q. 2) The distance travelled by an object in time t is given by $s = (2.5)t^2$. The instantaneous speed of the object at $t = 5s$ will be:- [JEE Mains 2024] ②

- (A) 5 m/s
- (B) 12.5 m/s
- (C) 62.5 m/s
- (D) 25 m/s

Ans:- $s = (2.5)t^2$

Differentiate,

$$\Rightarrow 2 \times 2.5 \times t$$

$$\Rightarrow 2 \times \frac{5}{2} \times t$$

$$s = 5t \quad [t = 5s]$$

$$\therefore s = (5)(5) \Rightarrow \underline{\underline{25 \text{ m/s}}} \quad (D)$$

Q. 3) A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3s and 5s is $\frac{x+1}{x}$. The value of x is. [$g = 10 \text{ m/s}^2$]. [JEE Mains 2024]

$$v = u - gt$$

$$v_1 = 150 - 10(3) \Rightarrow 120 \text{ m/s}$$

$$v_2 = 150 - 10(5) \Rightarrow 100 \text{ m/s}$$

$$\frac{v_1}{v_2} \Rightarrow \frac{120}{100} \Rightarrow \frac{12}{10} \Rightarrow \frac{6}{5} \times \frac{x+1}{x}$$

$$6x = 5x + 5$$

$$\therefore \underline{\underline{x = 5}}$$

Ex:- Motion in a Straight Line

Q1. A particle is moving in a straight line with an initial velocity of 2 m/s . It undergoes a constant acceleration of 3 m/s^2 for 4 seconds. Find the distance travelled by the particle in the last second of its motion.

Ans. Calculating the velocity ~~after~~ after 4 seconds using the equation $v = u + at$, where v is the final velocity, u is the initial velocity, a is the acceleration, and t is the time.

$$v = 2 + 3(4)$$

$$= 2 + 12$$

$$= \underline{14 \text{ m/s}}$$

Calculate the velocity at the start of the last second:

The particle accelerates for 4 seconds, so the start of the last second is at $t = 3$ seconds. Using

$v = u + at$ again:

$$v_{3s} = 2 + 3(3)$$

$$= 2 + 9$$

$$= \underline{11 \text{ m/s}}$$

Calculate the distance traveled in the last second.

The distance traveled in the last second can be calculated using the equation $s = vt + \frac{1}{2}at^2$, but

since we're considering only 1 second, it's easier to use $s = v_{\text{avg}} \times t$, where v_{avg} is the average velocity during that second.

$$V_{avg} = \frac{V_{3s} + V_{1s}}{2} = \frac{11 + 14}{2} = \frac{25}{2} = \underline{12.5 \text{ m/s}}$$

Distance traveled in the last second =

$$V_{avg} \times t = 12.5 \times 1 \\ = \underline{12.5 \text{ meters}}$$

Final answer = 12.5 meters

Q2. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come at rest. If the total time elapsed is t seconds, find the maximum velocity acquired by the car.

Ans. Let t_1 be the time the car accelerates, and t_2 be the time it decelerates. The total time is $t = t_1 + t_2$

The maximum velocity $v_{\max}$ is acquired at the end of the acceleration phase. Using $v = u + at$ for the acceleration phase:

$$v_{\max} = 0 + \alpha t_1 = \alpha t_1$$

Use the deceleration phase to relate t_1 and t_2 . During deceleration, the car comes to rest, so $v = 0$.

Using $v = u + at$ for the deceleration phase:

$$0 = v_{\max} - \beta t_2$$

$$v_{\max} = \beta t_2$$

Now,

$$\alpha t_1 = \beta t_2$$

Use the total time to find a relation between t_1 and t_2

$$t = t_1 + t_2$$

$$t_2 = t - t_1$$

Solving for t_1

$$\alpha t_1 = \beta (t - t_1)$$

$$\alpha t_1 = \beta t - \beta t_1$$

$$\alpha t_1 + \beta t_1 = \beta t$$

$$t_1 (\alpha + \beta) = \beta t$$

$$t_1 = \frac{\beta t}{\alpha + \beta} \quad t_2 = \frac{\alpha t}{\alpha + \beta}$$

Now, find v_{max}

$$v_{max} = \alpha t_1 = \frac{\alpha \beta t}{\alpha + \beta}$$

$$\underline{v_{max} = \frac{\alpha \beta t}{\alpha + \beta}}$$

$\therefore$ The final answer is $= \frac{\alpha \beta t}{\alpha + \beta}$

Q3. A particle moves in a straight line with a velocity given by $v(t) = 3t^2 - 2t + 1$ m/s. Find the displacement of the particle from $t = 0$ s to $t = 2$ s

Ans. We are given the velocity of a particle as a function of time, $v(t) = 3t^2 - 2t + 1$ m/s, and we need to find the displacement of the particle from $t = 0$ s to $t = 2$ s.

The displacement of an object can be found by integrating its velocity function with respect to time.

The displacement s from $t = 0$ s to $t = 2$ s is given by the integral:

$$s = \int_0^2 v(t) dt = \int_0^2 (3t^2 - 2t + 1) dt$$

Evaluating the integral, we get:

$$s = \left[\frac{3t^3}{3} - \frac{2t^2}{2} + t \right]_0^2$$

$\therefore$ The final answer is 6

Q.1. A person travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for next $\frac{3}{2}x$ distance. The average velocity in this motion is $\frac{50}{7}$ m/s. If v_1 is 5 m/s then $v_2 = \underline{\hspace{2cm}}$ m/s.

Ans.

Distance ' x ' at velocity $v_1 = 5$ m/s

Distance ' $\frac{3}{2}x$ ' at velocity v_2

$$\text{Average Velocity} = v_{\text{avg}} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{50 \text{ m/s}}{7} \quad [\text{Given}]$$

$$\text{Total Distance} = x + \frac{3x}{2} = \frac{5x}{2} \quad \left| \begin{array}{l} \text{Time for } x: t_1 = \frac{x}{v_1} = \frac{x}{5} \\ \text{Time for } \frac{3x}{2}: t_2 = \frac{\frac{3x}{2}}{v_2} = \frac{3x}{2v_2} \end{array} \right.$$

$$\Rightarrow \frac{50}{7} = \frac{\frac{5x}{2}}{\frac{x}{5} + \frac{3x}{2v_2}} = \frac{5/2}{\frac{1}{5} + \frac{3}{2v_2}}$$

$$\Rightarrow \frac{50}{7} = \frac{5/2}{\frac{1}{5} + \frac{3}{2v_2}}$$

$$\Rightarrow \frac{1}{5} + \frac{3}{2v_2} = \frac{7}{20}$$

$$\Rightarrow \frac{3}{2v_2} = \frac{3}{20}$$

$$\Rightarrow \underline{\underline{v_2 = 10 \text{ m/s}}}$$

Q.2. A body moves on a frictionless plane starting from rest. If S_n is distance moved between $t=n-1$ and $t=n$ and S_{n-1} is distance moved between $t=n-2$ and $t=n-1$, then the ratio $\frac{S_{n-1}}{S_n}$ is $(1-\frac{2}{n})$ for $n=10$. The value of $n = \underline{\hspace{2cm}}$?

Ans.

$$S = vt + \frac{1}{2}at^2 \quad [S = \frac{1}{2}at^2 \text{ as } v=0]$$

The distance moved between $t=n-1$ & $t=n$ (S_n) can be found by calculating distance covered by end of time n & subtracting the distance covered by end of time $(n-1)$. Let's denote total distance covered by time n as $S(n)$, which, acc. to formula: - $S(n) = \frac{1}{2}an^2$
 $\Rightarrow S_n = S(n) - S(n-1)$

$$\Rightarrow S_n = \frac{1}{2}an^2 - \frac{1}{2}a(n-1)^2 = \frac{1}{2}a(n^2 - (n^2 - 2n + 1))$$

$$\Rightarrow S_n = \frac{1}{2}a(2n-1)$$

$$\text{Similarly } S_{n-1} = \frac{1}{2}a((n-1)^2 - (n-2)^2) = \frac{1}{2}a(n^2 - 1^2 - (n^2 - 4n + 4))$$

$$\Rightarrow S_{n-1} = \frac{1}{2}a(n^2 - 2n + 1 - n^2 + 4n - 4)$$

$$\Rightarrow S_{n-1} = \frac{1}{2}a(2n-3)$$

$$\Rightarrow \frac{S_{n-1}}{S_n} = \frac{\frac{1}{2}a(2n-3)}{\frac{1}{2}a(2n-1)} = \frac{2n-3}{2n-1}$$

$$\text{For } n=10, \frac{S_{n-1}}{S_n} = \frac{17}{19}$$

$$\text{Given, ratio is represented by } 1 - \frac{2}{n} \Rightarrow \frac{17}{19} = 1 - \frac{2}{n}$$

$$\Rightarrow \frac{2}{n} = \frac{2}{10} \Rightarrow \underline{\underline{n=10}}$$

Q3 A car starts from rest and accelerates uniformly to a speed of 72 km/hr in 10s. It then moves with a uniform speed for 20s and is finally brought to rest in 5s with constant retardation. Find the total distance covered by the car.

Ans. $72 \text{ km/hr} = 72 \times \frac{5 \text{ m/s}}{18} = \underline{20 \text{ m/s}}$

$$\text{Acceleration} = \frac{v-u}{t} = \frac{20}{10} = \underline{2 \text{ m/s}^2}$$

$$\begin{aligned} \text{Distance covered during acceleration} = S_1 &= ut + \frac{1}{2}at^2 \\ &= 0 + \frac{1}{2}(2)(100) = \underline{100 \text{ m}} \end{aligned}$$

$$\text{Distance covered during uniform motion} = S_2 = vt = 20 \times 20 = \underline{400 \text{ m}}$$

$$\text{Retardation of car} = \frac{v-u}{t} = \frac{-20}{5} = \underline{-4 \text{ m/s}^2}$$

$$\begin{aligned} \Rightarrow \text{Distance covered during retardation} = S_3 &= ut + \frac{1}{2}at^2 \\ &= 20(5) + \frac{1}{2}(-4)(25) \\ &= 100 - 50 = \underline{50 \text{ m}} \end{aligned}$$

$$\begin{aligned} \therefore \text{Total Distance} &= 100 + 400 + 50 \\ &= \underline{550 \text{ m}} \end{aligned}$$

Motion In A Straight line

Q1) Parachutist jumps out of plane and accelerates with gravity for 6 sec. He then pulls parachute chord and after 4s deceleration period, descends at 10 m/s for 60 seconds, reaching ground. From what height did parachutist jump? Assume acceleration due to gravity to be 10 m/s^2 throughout motion.

Solution:

Ist motion: Person jumped for 6 seconds with gravity.

$$s_0, a = g = 10 \text{ m/s}^2$$

$u = 0$ (no initial velocity as plane moved horizontal)

$$t = 6 \text{ s}$$

$$s = ut + \frac{1}{2}at^2$$

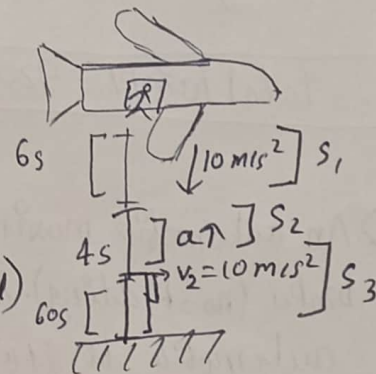
$$s = 0 \times 6 + \frac{1}{2} \times 10 \times 6^2$$

$$s_1 = 0 + 180 = 180 \text{ m}$$

$$v = u + at$$

$$v = 0 + 10 \times 6$$

$$v_1 = 60 \text{ m/s}$$



IInd motion: person opened parachute and descended for 4 ~~ms~~ second:

$$s_0, a = ?$$

$u = 60 \text{ m/s}$ (person was at $v = 60 \text{ m/s}$ before opening parachute)

$v_2 = 10 \text{ m/s}$ (after 4 seconds of deceleration, he moved with uniform $v = 10 \text{ m/s}$)

$$t = 4 \text{ s}$$

$$v = u + at$$

$$10 = 60 + a \times 4$$

$$4a = 10 - 60 = -50 \Rightarrow a = -\frac{25}{2} \text{ m/s}^2$$

$$s = ut + \frac{1}{2}at^2$$

$$s_2 = 60 \times 4 - \frac{1}{2} \times \frac{25}{2} \times 4 \times 4$$

$$s_2 = 240 - 100 = 140 \text{ m}$$

IIIrd motion: Person moving with uniform velocity for 60 seconds

$$V = 10 \text{ m/s}$$

$$t = 60 \text{ s}$$

$$S = Vt \text{ (uniform velocity)}$$

$$S_3 = 60 \times 10$$

$$S_3 = 600 \text{ m}$$

$$\text{Total height } H = S_1 + S_2 + S_3 = (140 + 180 + 600) \text{ m} \\ = 920 \text{ m}$$

$$\therefore \text{total height} = 920 \text{ m}$$

Q) An automobile moving at 40 km/h can be stopped in 40 m using brakes (no skidding). At what minimum distance can same automobile be stopped if it's moving at 80 km/h?

(JEE Main-2018)

(a) 75 m (b) 100 m (c) 150 m (d) 160 m

Q) At $u = 40 \text{ km/h}$, $v = 0$ (body comes to rest) $S = 40 \text{ m} = 0.04 \text{ km}$

$$2as = v^2 - u^2$$

$$2 \times a \times 0.04 = 0 - (40)^2$$

$$a = \frac{-1600}{0.08} = \frac{-160000}{8} = -20,000 \text{ km/h}^2$$

At $u = 80 \text{ km/h}$, $v = 0$ (body comes to rest), $a = -20,000 \text{ km/h}^2$ (uniform acceleration)

$$2as = v^2 - u^2$$

$$2 \times -20,000 \times S = 0^2 - (80)^2$$

$$S = \frac{+6400}{-40000} = \frac{16}{100} = 0.16 \text{ km} = 160 \text{ m}$$

$$\therefore \text{minimum distance} = 160 \text{ m}$$

$$(d) 160 \text{ m}$$

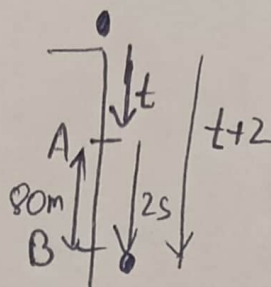
Q) A body falling under gravity covers two points A and B separated by 80 m in 2 s. The distance of upper point A from starting point is _____ m (use $g = 10 \text{ m s}^{-2}$)

Sol:

t = time taken to reach point A

$u = 0$ { body dropped from rest }

$a = g = 10 \text{ m/s}^2$



$$S_t = ut + \frac{1}{2} a t^2$$

$$S_t = 0 + \frac{1}{2} \times 10 \times t \times t$$

$$S_t = 5t^2$$

$t+2$ → { time taken to reach point B }

$a = g = 10 \text{ m/s}^2$

$u = 0$ { body dropped }

$$S = ut + \frac{1}{2} a t^2$$

$$S_{t+2} = 0 + \frac{1}{2} \times 10 \times (t+2)^2$$

$$S_{t+2} = 5(t^2 + 2t + 4)$$

$$S_{t+2} = 5t^2 + 20t + 20$$

$$S_{t+2} - S_t = S_2 \text{ { displacement in 2 sec } = S_2 = 80 \text{ m } }$$

$$5t^2 + 20t + 20 - 5t^2 = 80$$

$$20t + 20 = 80$$

$$t + 1 = 4$$

$$\boxed{t = 3 \text{ s}}$$

At time $t = 3 \text{ s}$, body reaches A

$$S_A = ut + \frac{1}{2} a t^2$$

$$S_A = 0 + \frac{1}{2} \times 10 \times 3 \times 3$$

$$\boxed{S_A = 45 \text{ m}}$$

Ans: 45 m

2025-26

Motion in a Straight Line

Q1) A particle moves along the x-axis and has its displacement x varying with time t according to the equation:

$$x = C_0 (t^2 - 2) + C (t - 2)^2$$

[Where, C_0 and C are constant of appropriate dimension.]

Then, which of the following statements is correct?

- Ⓐ The acceleration of the particle is $2(C + C_0)$
- Ⓑ The acceleration of the particle is $2C_0$.
- Ⓒ The acceleration of the particle is $2C$.
- Ⓓ The initial velocity of the particle is $4C$.

Ans:- Given,

$$x = C_0 (t^2 - 2) + C (t - 2)^2 \quad [x \rightarrow \text{displacement}]$$

$$\therefore v = \frac{dx}{dt} = \frac{d}{dt} [C_0 (t^2 - 2) + C (t - 2)^2]$$

$$= \frac{d}{dt} \{C_0 (t^2 - 2)\} + \frac{d}{dt} \{C (t - 2)^2\}$$

$$= C_0 (2t) + C (2t - 2) \Rightarrow v = C_0 (2t) + C (2t - 2)$$

$$\therefore a = \frac{dv}{dt} = \frac{d}{dt} \{C_0 (2t) + C (2t - 2)\} = 2C_0 + 2C$$

$$\Rightarrow a = 2(C_0 + C)$$

$\therefore$ Acceleration is $2(C_0 + C)$ [Ⓐ]

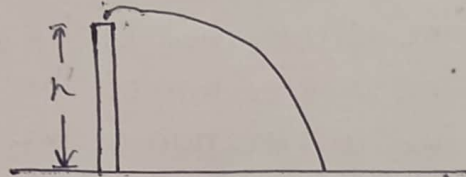
② A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1 . If it is projected vertically downwards from the same point with the same speed, it reaches the ground in t_2 . Time required to reach the ground, if it is dropped from the top of the tower is:

- (A) $\sqrt{t_1 + t_2}$ (B) $\sqrt{t_1 t_2}$ (C) $\sqrt{\frac{t_1}{t_2}}$ (D) $\sqrt{t_1 - t_2}$

Ans:-

CASE 1:-

$$h = -ut_1 + \frac{1}{2}gt_1^2 \quad \dots (i)$$



CASE 2:-

$$h = ut_2 + \frac{1}{2}gt_2^2 \quad \dots (ii)$$

$\therefore$ From (i), (ii), we get

$$-ut_1 + \frac{1}{2}gt_1^2 = ut_2 + \frac{1}{2}gt_2^2$$

$$\Rightarrow -u(t_1 + t_2) = \frac{1}{2}g(t_2^2 - t_1^2)$$

$$\Rightarrow -u = \frac{1}{2}g(t_2 - t_1)$$

$$\Rightarrow u = \frac{1}{2}g(t_1 - t_2) \quad \dots (iii)$$

Substituting (iii) in (i), we get

$$\Rightarrow h = -\left\{\frac{1}{2}g(t_1 - t_2)\right\}t_1 + \frac{1}{2}gt_1^2$$

$$= \frac{1}{2}g(t_2 t_1 - t_1^2) + \frac{1}{2}gt_1^2$$

$$\Rightarrow h = \frac{1}{2}g(t_2 t_1) - \frac{1}{2}gt_1^2 + \frac{1}{2}gt_1^2$$

$$\Rightarrow h = \frac{1}{2}g(t_2 - t_1) \quad \dots (iv)$$

CASE 3:-

$$h = \frac{1}{2}gt^2 \quad \dots (v)$$

[$\because$ free fall]

So, $u = 0$

$$\therefore \frac{1}{2}gt^2 = \frac{1}{2}g(t_2 - t_1)$$

$$\Rightarrow t = \sqrt{t_2 - t_1} \quad [D]$$

$\therefore$ Required time is,
 $t = \sqrt{t_2 - t_1} \quad [D]$

③ A train starting from rest first accelerated uniformly up to a speed of 80 kmph, for time t then it moves with a constant speed of time $3t$. The avg. speed of the train for this duration of journey (kmph):

- Ⓐ 70 Ⓑ 40 Ⓒ 30 Ⓓ 80

Ans:-

$$u_1 = 0; v_1 = 80 \text{ kmph}; t_1 = t.$$

$$v_1 = u_1 + at_1$$

$$\Rightarrow 80 = 0 + at_1 \Rightarrow t_1 = \frac{80}{a} \quad \text{--- (i)}$$

$$S_1 = ut_1 + \frac{1}{2}at_1^2 \Rightarrow S_1 = 0 + \frac{1}{2} \cdot a \cdot \frac{80}{a} \cdot \frac{80}{a} \Rightarrow S_1 = \frac{3200}{a} \quad \text{--- (ii)}$$

$$\therefore u_2 = 80 \text{ kmph}; v_2 = ?; a = 0; t_2 = 3t$$

$$\therefore t_2 = 3 \times \frac{80}{a} \Rightarrow t_2 = \frac{240}{a}$$

$$\therefore u_2 + at_2 = v_2$$

$$\Rightarrow u_2 + 0 = v_2 \Rightarrow v_2 = u_2 = 80 \text{ kmph}$$

$$\therefore S = ut + \frac{1}{2}at^2 \Rightarrow S_2 = 80(3t) \Rightarrow 240t = S_2$$

$$\Rightarrow S_2 = 240 \times \frac{80}{a} \Rightarrow S_2 = \frac{19200}{a} \quad \text{--- (iii)}$$

$$\text{Avg. Speed} = \frac{S_1 + S_2}{t_1 + t_2} = \frac{3200 + 19200}{\frac{80}{a} + \frac{240}{a}} \times \frac{a}{80 + 240}$$

$$\Rightarrow \text{Avg. Speed} = 70 \text{ kmph} \quad \text{Ⓐ}$$

1. A body moves 6m north, 8m east and 10m vertically upwards what is its resultant displacement from initial position

(only magnitude) (a) $10\sqrt{2}$ m (b) 10m (c) $10/\sqrt{2}$ m (d)

Answer :- $dx = 8\text{m (east)}$ $dy = 6\text{m (north)}$ $dz = 10\text{m (upward)}$

$$|\vec{d}| = \sqrt{dx^2 + dy^2 + dz^2}$$

$$\sqrt{8^2 + 6^2 + 10^2}$$

$$\Rightarrow \sqrt{64 + 36 + 100}$$

$$\Rightarrow \sqrt{200}$$

$$= 10\sqrt{2}\text{m},$$

2. A small toy starts moving from the position of rest under a constant acceleration. If it travels a distance of 10m in t s the distance travelled by the toy in next t s will be

Answer :- $S = ut + \frac{1}{2}at^2$, $u=0$

$$S_1 = 10\text{m} \quad S_1 = \frac{1}{2}at^2$$

$$10 = \frac{1}{2}at^2 = at^2 = 20 - 0$$

$$S_2 = \frac{1}{2}a(2t)^2 = \frac{1}{2}a(4t^2) = 2at^2$$

$$S_2 = 2 \times 20 = 40\text{m}$$

$$S = S_2 - S_1 = 40 - 10 = 30\text{m}$$

3. A ball of mass 0.5 kg is dropped from the height of 10 m . The height, at which the magnitude of velocity becomes equal to the magnitude of acceleration due to gravity, is m ($g=10\text{ m/s}^2$)

Answer:- $m=0.5\text{ kg}$ $h_0=10\text{ m}$ $g=10\text{ m/s}^2$ $v=10$

$$v^2 = u^2 + 2gs$$

$$(+10)^2 = 0^2 + 2(10)s$$

$$100 = 20s$$

$$s = \frac{100}{20} = 5\text{ m}$$

Q.1 A ball is dropped from the top of a 100m high tower on a planet. In the last $\frac{1}{2}$ s before hitting the ground, it covers, a distance of 19m. Acceleration due to gravity (g) (in ms^{-2}) near the surface on that planet is.

Ans: Let the ball takes time t to reach the ground.

$$\text{Using, } S = ut + \frac{1}{2}gt^2$$

$$S = 0 + \frac{1}{2}gt^2$$

$$S = \frac{1}{2}gt^2$$

$$200 = gt^2$$

$$t = \sqrt{\frac{200}{g}}$$

In last, $\frac{1}{2}$ s body travels a distance of 19m, so in $(t - \frac{1}{2})$

Distance travelled = 81

$$\frac{1}{2}g\left(t - \frac{1}{2}\right)^2 = 81$$

$$g\left(t - \frac{1}{2}\right) = 81 \times 2$$

$$\frac{1}{2} = \frac{1}{\sqrt{g}} (\sqrt{200} - \sqrt{81 \times 2})$$

$$\sqrt{g} = 2(10\sqrt{2} - 9\sqrt{2})$$

$$\sqrt{g} = 2\sqrt{2}$$

$$g = 8 \text{ m/s}^2$$

Q.2 The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ and $t = 0$, then its displacement after unit time ($t = 1$) is

Ans: We know that $v = \frac{dx}{dt}$

$$dx = v dt$$

$$\text{Integrating } \int_0^x dx = \int_0^t v dt \text{ or } x = \int_0^t (v_0 + gt + ft^2) dt = \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3} \right]$$

$$\text{or } x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$$

$$\text{At } t = 1, x = v_0 + \frac{g}{2} + \frac{f}{3}$$

Q.3 A particle located at $x = 0$ and time $t = 0$, starts moving along with the positive x -direction with a velocity 'v' that varies as $v = a\sqrt{x}$. The displacement of the particle varies with time as.

$$\text{Ans: } v = a\sqrt{x} \text{ or } \frac{dx}{dt} = a\sqrt{x} \text{ or } \frac{dx}{\sqrt{x}} = a dt$$

$$\text{Integrating both sides, } \int_0^x \frac{dx}{\sqrt{x}} = a \int_0^t dt ; \left[\frac{2\sqrt{x}}{1} \right]_0^x = a[t]_0^t$$

$$= 2\sqrt{x} = at = x = \frac{a^2}{4} t^2$$

Motion in a straight line.

Q1. Two cars travel towards each other at 20 m/s each when they are 300 m apart, both apply brakes and retard at 2 m/s^2 . The distance between them when they come at rest.

Sol:

$$\text{Relative speed} = 20 + 20 = 40 \text{ m/s}.$$

$$\text{Relative retardation} = 2 + 2 = 4 \text{ m/s}^2.$$

$$\text{Stopping distance: } s = \frac{v^2}{2a} = \frac{40^2}{2 \times 4} = \frac{1600}{8} = 200 \text{ m}$$

$$\text{Remaining gap} = 100 \text{ m}$$

Q2. Particle covers half the distance at speed 6 m/s the remaining half is covered in two equal time intervals at speeds 9 m/s and 15 m/s . avg speed = ?

Sol:

$$\text{First half: } T_1 = \frac{P/2}{6} = \frac{P}{12}$$

$$\text{Distance} = 9T + 15T = 24T = \frac{P}{2}.$$

$$T = \frac{P}{48} \Rightarrow t_2 = 2T = \frac{P}{24}.$$

$$\text{Total time} = \frac{P}{12} + \frac{P}{24} = \frac{P}{8}.$$

$$\text{Avg Speed} = \frac{P}{P/8} = 8 \text{ m/s}$$

Q3. for a particle, the relation between T and x given by $T = \alpha x^2 + \beta x$.

find relation between acceleration "a" and velocity "v"

Solu:

$$\frac{dt}{dx} = 2\alpha x + \beta \Rightarrow v = \frac{dx}{dt} = \frac{1}{2\alpha x + \beta}$$

$$\frac{dv}{dx} = \frac{-2\alpha}{(2\alpha x + \beta)^2}$$

$$a = v \frac{dv}{dx} = \frac{1}{2\alpha x + \beta} \times \frac{-2\alpha}{(2\alpha x + \beta)^2} = \frac{-2\alpha}{(2\alpha x + \beta)^3}$$

$$a = -2\alpha v^3$$

CP=

- Q. A person of $h = 1.6\text{m}$ is walking away from a lamp post 4m along a straight path on the flat ground. The lamp post & the person are always perpendicular to the ground. If the speed of the person is 60cm s^{-1} , the speed of the tip of the person's shadow on the ground with respect to the person is cm s^{-1} .

Sol: $\frac{dx_1}{dt}$ = speed of person = 60cm/s
 $\frac{dx_2}{dt}$ = speed of tip of person's shadow

Apply $\sim \Delta$ rule in $\triangle ABE$ & $\triangle DCE$,

$$\frac{4}{x_2} = \frac{1.6}{x_2 - x_1}$$

$$4x_2 - 4x_1 = 1.6x_2$$

$$2.4x_2 = 4x_1$$

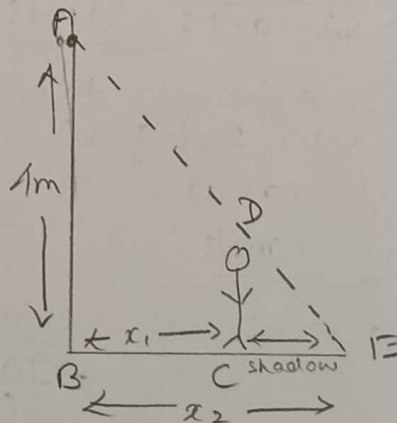
Differentiating both sides:

$$2.4 \frac{dx_2}{dt} = 4 \frac{dx_1}{dt}$$

$$\frac{dx_2}{dt} = \frac{4}{2.4} (60) = 100\text{ cm/s}$$

$$\vec{V}_{sp} = \vec{V}_{s\&} - \vec{V}_{P\&}$$

$$V_{sp} = 100 - 60 \\ = \underline{\underline{40\text{ cm s}^{-1}}}$$



- Q. A force of 40N is responsible for the motion of a body governed by the eq. $s = 2t + 2t^2$ where s is in meters & t in sec. What is the momentum of the body at $t = 2\text{sec}$?

Sol: $P = ?$ at $t = 2\text{secs}$ $F = 40\text{N}$, $s = 2t + 2t^2$

(i) $v = \frac{d}{dt}(s)$ $v = \frac{d}{dt}(2t + 2t^2)$
 $v = 2 + 4t$

$a = \frac{d}{dt}(2 + 4t) \therefore \underline{a = 4\text{m/s}^2}$

$F = ma$, $F = 40$

$40 = 11a$

$m = F/a$

$m = 40/4 = \boxed{10\text{kg}}$

$v = 2 + 4t$

At $t = 2\text{secs}$

$v = 2 + 4(2)$

$v = 10\text{m/s}$

P at $t = 2\text{secs}$

$P = 10 \times 10$

$\boxed{P = 100\text{kg m/s}}$

- Q. A tennis ball dropped on to the floor from a height of 9.8m. It rebounds to a height 5m. Ball comes in contact with the floor for 0.2secs. The avg. acceleration during contact is:

$v^2 = 2gh$

$v = \sqrt{2gh}$

$v = \sqrt{2 \times 10 \times 9.8} = \underline{14\text{m/s}}$

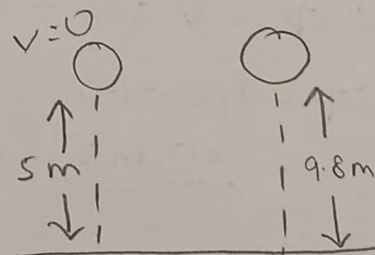
Ball rebounds to $h = 5\text{m}$

$0 = v_1^2 + 2 \times (-g) \times 5$

$v_1^2 = 10 \times 10$

$\underline{v_1 = 10\text{m/s}}$

Avg acceleration = $\frac{v_1 - (v)}{t}$
 $= \frac{10 + 14}{0.2} = \frac{24}{0.2} = 120\text{m/s}^2$



Q.) A body projected vertically upwards with a certain speed from top of a tower reaches the ground in t_1 . If it's projected vertically downwards from the same point with the same speed, it reaches the ground t_2 . Time required to reach the ground, if it is dropped from top of the tower is:

(A) $\sqrt{t_1 t_2}$

(B) $\sqrt{t_1 + t_2}$

(C) $\sqrt{t_1 - t_2}$

(D) $\sqrt{\frac{t_1}{t_2}}$

Ans $\rightarrow t_1 = \frac{u + \sqrt{u^2 + 2gh}}{g}$

$t_2 = \frac{-u + \sqrt{u^2 + 2gh}}{g}$

$t = \frac{\sqrt{2gh}}{g}$

$t_1 t_2 = \frac{(u^2 + 2gh) - u^2}{g^2} = \frac{2gh}{g^2} = t^2$

$t = \sqrt{t_1 t_2}$

Q.) The velocity of a particle is given $\vec{v} = -x\hat{i} + 2y\hat{j} - z\hat{k}$ m/s.
The magnitude of $\vec{a}$ at (1, 2, 4) is ms^{-2} .

(A) $\sqrt{6}$

(C) 9

(B) $\sqrt{33}$

(D) 0

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = -x\hat{i} + 2y\hat{j} - z\hat{k}$$

$$a_x = (-x)(-1) = x$$

$$a_y = 2y(2) = 4y$$

$$a_z = -z(-1) = z$$

$$\vec{a} = \hat{i} + 8\hat{j} + 4\hat{k}$$

$$|\vec{a}| = 9 \text{ ms}^{-2}$$

Q.) A particle moves in a straight line so that its displacement 'x' at any time 't' is given by $x^2 = 1 + t^2$. Its acceleration at any time 't' is x^{-n} where $n = \underline{\hspace{2cm}}$

Ans $\rightarrow$

$$x^2 = 1 + t^2$$

$$2x \frac{dx}{dt} = 2t$$

$$x \cdot v = t$$

$$x \frac{dv}{dt} + v \frac{dx}{dt} = 1$$

$$xa + v^2 = 1$$

$$a = \frac{1}{x^3} = x^{-3}$$

CH-2 MOTION IN A STRAIGHT LINE

Q1 A train is moving in a straight line with constant acceleration a . A boy standing inside throws a ball forward at speed 10 m/s at angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train in order to catch the ball back at same height. Find acc. of the train (in m/s^2) [JEE Adv. 2011].

$\Rightarrow$ Given : $u = 10 \text{ m/s}$; $\theta = 60^\circ$

Hori. component :

$$\begin{aligned} u_x &= u \cos \theta \\ &= 10 \cos(60^\circ) \\ &= \underline{\underline{5 \text{ m/s}}} \end{aligned}$$

Vertical component :

$$\begin{aligned} u_y &= u \sin \theta \\ &= 10 \times \frac{\sqrt{3}}{2} \\ &= \underline{\underline{5\sqrt{3} \text{ m/s}}} \end{aligned}$$

$$T = \frac{2u_y}{g} = \frac{2(5\sqrt{3})}{10} = \underline{\underline{\sqrt{3}}}$$

Disp. of ball : $V_0 \Rightarrow$ initial velo. of train

hori. dist covered by ball at $T = \sqrt{3}$:

$$x_{\text{ball}} = (V_0 + 5) \cdot \sqrt{3}$$

Disp of boy :

$$x_{\text{boy}} = \cancel{u_0} t + \frac{1}{2} a t^2$$

$$= u_0 \cdot \sqrt{3} + \frac{1}{2} a (\sqrt{3})^2$$

$$= u_0 \cdot \sqrt{3} + \frac{3a}{2}$$

Now Relative disp = 1.15

$$\therefore x_{\text{ball}} - x_{\text{boy}} = 1.15$$

$$= (u_0 + 5)\sqrt{3} - u_0 \cdot \sqrt{3} + \frac{3a}{2} = 1.15$$

$$= 5\sqrt{3} - \frac{3a}{2} = 1.15$$

$$\text{Acc} = - \frac{3a}{2} = 1.15 - 5\sqrt{3}$$

$$= - \frac{3a}{2} = -7.51$$

$$a = \frac{2(7.51)}{3}$$

$$a = 5.006 \text{ m/s}^2$$

$$\boxed{a = 5 \text{ m/s}^2}$$

Q2 A particle moves in a straight line such that its displ. is given by

$$x = t^3 - 4t^2 + 3t$$

Find the acceleration of the particle at the times when its displacement is zero.

$$\begin{aligned} \Rightarrow \frac{dx}{dt} (t^3 - 4t^2 + 3t) \\ = 3t^2 - 8t + 3 \end{aligned}$$

velo.

$$\frac{d}{dt} (3t^2 - 8t + 3)$$

acc.

$$= 6t - 8 \quad \text{---} \quad \textcircled{1}$$

Factor of $t^3 - 4t^2 + 3t$

$$= t(t^2 - 4t + 3)$$

$$= t[(t-1)(t-3)]$$

$$\therefore t = 0 ; t = 3 \quad \text{---} \quad \textcircled{2}$$

$t = 1$

② in 1

$$t = 0 \quad \rightarrow \quad a = -8 //$$

$$t = 1 \quad \rightarrow \quad a = -2 //$$

$$t = 3 \quad \rightarrow \quad a = 10 //$$

Q A particle starts from rest and moves in a straight line with acc :

$$a(t) = 6t - 12$$

(a is m/s^2 and t is sec)

- Find time at which particle comes to rest.
- Find total dist. travelled by particle in first 4 sec.
- Find the displacement in first 4 sec.

$$\Rightarrow a(t) = \frac{dv}{dt} = 6t - 12$$

$$\int a(t) \cdot dt \Rightarrow \int (6t - 12) \cdot dt$$
$$\Rightarrow 3t^2 - 12t + C$$

as we start from rest $\Rightarrow v = 0$
 $\therefore C = 0$

$$v(t) = 3t^2 - 12t$$

$$v(t) = 0 \Rightarrow 3t^2 - 12t = 0$$

$$3t(t - 4) = 0$$

$$t = 0 \text{ or } t = 4$$

$\therefore$ particle comes at rest again at

$$\underline{t = 4 \text{ sec}} \longrightarrow \underline{(a)} \longrightarrow \underline{\text{Ans}}$$

Total distance is first 4 sec

$$v(t) = 3t(t-4)$$

$v(t) > 0$ when $0 < t < 4$

$$v(0) = 0 ; v(4) = 0$$

veloc. is positive.

$$s(t) = \int (v(t)) dt$$

$$= \int 3t^2 - 12t dt$$

$$= t^3 - 6t^2 + C$$

$$s(t) = t^3 - 6t^2$$

Now, $t = 4$

$$s(4) = (4)^3 - 6(4)^2$$

$$= 64 - 96$$

$$= -32\text{m}$$

$\therefore$ dist. traveled in first 4 sec = 32m

$\rightarrow (b) \rightarrow \text{Ans.}$

From $v(t) = 3t(t-4)$

motion is backward from $t=0$ to $t=4$

$\therefore$ displacement = -32m

$\rightarrow (c) \rightarrow \text{Ans}$